

# AN EXPERIMENTAL DETERMINATION OF THE HEAT AND MASS TRANSFER COEFFICIENTS IN MOIST, UNSATURATED SOILS

D. J. SHAH,<sup>†</sup> J. W. RAMSEY and M. WANG<sup>‡</sup>

Department of Mechanical Engineering, University of Minnesota, 111 Church Street Southeast, Minneapolis, MN 55455, U.S.A.

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**Abstract**—Ratios of the thermal diffusion coefficient to the moisture diffusion coefficient were experimentally determined for two types of soils. The ratio of the diffusion coefficients was found to increase with increasing moisture content, reach a broad maximum, and thereafter decrease. Except for the drier regions near the warm end, the ratio was found to be in the range  $10^{-3}$ – $10^{-2}$  °C<sup>-1</sup>. Analytical predictions for the thermal diffusion coefficients were combined with the experimental results to determine the moisture diffusion coefficients. The thermal diffusion coefficient was predicted to decrease with both decreasing temperature and increasing moisture content. For the soils used in this study, the moisture diffusion coefficient was estimated to be in the range  $10^{-5}$ – $10^{-4}$  g s<sup>-1</sup> cm<sup>-1</sup>.

## NOMENCLATURE

|           |                                                                                            |
|-----------|--------------------------------------------------------------------------------------------|
| $C_s$     | specific heat of moist soil (based on per unit mass of dry soil) (energy/mass temperature) |
| $D$       | diffusion coefficient of water vapor into air (area/time)                                  |
| $D^*$     | thermal (liquid + vapor) diffusion coefficient (mass/time length temperature)              |
| $D_l^*$   | thermal diffusion coefficient of liquid water (mass/time length temperature)               |
| $D_v^*$   | thermal diffusion coefficient of water vapor (mass/time length temperature)                |
| $f$       | porosity                                                                                   |
| $J_l$     | mass flux of liquid water (mass/time area)                                                 |
| $J_{net}$ | net moisture flux (mass/time area)                                                         |
| $J_v$     | mass flux of water vapor (mass/time area)                                                  |
| $K$       | moisture (liquid + vapor) diffusion coefficient (mass/time length)                         |
| $K_l$     | moisture diffusion coefficient of liquid water (mass/time length)                          |
| $K_v$     | moisture diffusion coefficient of water vapor (mass/time length)                           |
| $k_s$     | apparent thermal conductivity of moist soil (energy/length temperature)                    |
| $L$       | length of soil sample                                                                      |
| $T$       | temperature                                                                                |
| $T_C$     | temperature at the cold end of the test cell                                               |
| $T_H$     | temperature at the warm end of the test cell                                               |
| $W$       | soil moisture content (mass of liquid water/mass of dry soil)                              |
| $W_i$     | initial moisture content of the soil (mass of liquid water/mass of dry soil)               |
| $W_s$     | saturation moisture content of the soil (mass of liquid water/mass of dry soil)            |
| $w$       | mass fraction of water vapor (mass of vapor/mass of mixture)                               |

$x$  axial distance from the warm end of the test cell (length).

## Greek symbols

|               |                                                                                                       |
|---------------|-------------------------------------------------------------------------------------------------------|
| $\varepsilon$ | tortuosity (taken as 0.65 in the current study)                                                       |
| $\theta$      | dimensionless temperature, $(T - T_C)/(T_H - T_C)$                                                    |
| $\rho_a$      | density of saturated air (mass/volume)                                                                |
| $\rho_d$      | density of dry soil (mass/volume)                                                                     |
| $\rho_v$      | density of water vapor (assumed to be saturated vapor for all locations where $W > 0$ ) (mass/volume) |
| $\tau$        | time.                                                                                                 |

## INTRODUCTION

THE HEATING and cooling requirements of a conventional above-ground structure are predominantly effected by the seasonal variations of the ambient conditions to which the building is exposed. Due to escalating energy costs and concern for the environment, aesthetic buildings are being blended into the surroundings by constructing earth-sheltered structures. In an above-ground structure, it is necessary to use insulation to increase the thermal resistance of the building in order to decrease the energy requirements for space conditioning. However, in a sub-surface construction, such actions are not always necessary since the surrounding soil provides some resistance to heat loss. Furthermore, the temperature difference between the building and the soil is generally less than the temperature difference between the building and the ambient, thus resulting in a reduction in the energy required to maintain the interior of the building at the desired temperature. In order to evaluate the effectiveness of the soil in insulating a building, it is necessary to know the thermal conductivity of the soil. This thermal conductivity is strongly dependent on the moisture content of the soil which in turn, is a function of the temperature profile and the moisture transport

<sup>†</sup> Present address: Ellerbe Associates, Inc., Bloomington, MN 55420, U.S.A.

<sup>‡</sup> Present address: Huadong Petroleum Institute, Dong Ying, Shandong Province, People's Republic of China.

properties. Thus, a complete analysis of heat transfer from an earth-sheltered building requires the knowledge of the individual heat and mass transfer coefficients of the soil.

Chemical and petroleum engineers are also showing interest in this area because of concern regarding the depletion of crude oil reserves. In order to remove all the crude oil from a well, enhanced recovery techniques are being utilized, where a detergent or steam is used to dry-clean the oil well. Analysis of this process and numerous other applications (e.g. using the ground as a heat source and/or sink, as a storage medium, etc.) also requires the knowledge of the transfer properties of soils.

Until recently, it was the soil scientists and the utility companies that were primarily interested in the properties of soils. Researchers in soil engineering have presented experimental and analytical results indicating the effect of solar radiation, seasonal ambient temperature variations, and underground utility lines on the mass flux of moisture in the root zones of plants. The conclusions from these studies were of interest to the utility companies for understanding the effects of soil type on the temperature of buried electrical cables, natural gas in pipelines, and domestic water pipes.

The coupled process of heat and mass transfer in moist soils was not fully described until 1957. Hence the conclusions drawn by some of the early pioneers in this field were not always correct. However, many of their experimental observations provided valuable information used by subsequent investigators in formulating the currently accepted heat and moisture transfer models.

While investigating the thermal conductivity of moist soils, Smith [1] observed that there was a net transfer of moisture from the warm to the cold end of cylindrical soil samples, which he attributed to transfer in the vapor phase. In a later paper, Smith [2] presented data showing the transfer of moisture due to temperature gradients for several soils with different initial moisture contents. Smith [2] was apparently the first to propose the following mechanism for thermally induced moisture movement in soils: When a temperature gradient is established in a soil sample, water begins to evaporate in the warm region and begins to condense just ahead in the cooler region. Due to a difference in the soil moisture content, liquid water moves from the cold to the warm section by capillarity. This process, which was triggered by vapor movement, will continue until all the moisture in the warm section, except that held in place by surface tension in the form of small rings around the soil grains, has moved to the colder section of the soil.

Phillip and deVries [3] are often credited with being the first to correctly describe the coupling of heat and mass transfer in moist soil: When a soil dries and liquid continuity is disrupted, some liquid water still remaining in the soil tends to be located in the small pore spaces between the soil particles. This forms islands of water between the soil particles, which are distinct from

the absorbed water on the surface of the particles. Water vapor condenses on the cooler side of the island and evaporates from the other, relatively warmer, side. The moisture flux is then determined by the gradient of vapor pressure across the air-filled pores.

A theoretical model describing the coupled heat and mass transfer process in moist soils also was first presented by Phillip and deVries [3]. This model is based on the classical concepts of heat and mass transfer in porous media under combined temperature and moisture gradients [4, 5]. The approach requires that the physical properties of the porous system affecting both heat and moisture movement be known precisely so that reliable estimates can be made of the individual diffusion coefficients which appear in the transport equations.

Based on the information available from past research, it is possible to draw the following conclusions regarding heat and mass transfer in moist soils: Heat and mass transfer in moist soils is a coupled process. Moisture moves in the form of water vapor from the warm to the cold soil while liquid water transfer takes place in the opposite direction. The mass flux of water vapor is primarily due to the temperature gradient, while the mass flux of liquid water is primarily due to the difference in the soil moisture content. Under steady-state conditions, the mass flux of water vapor is equal and opposite in direction to the mass flux of liquid water. In order to accurately predict the soil moisture content, it is necessary to know the soil diffusion coefficients for the mass flux of water vapor and the mass flux of liquid water.

The objective of the present investigation was to experimentally determine the soil diffusion coefficients for two types of soils. The necessary equations for obtaining the diffusion coefficients, the design of an experimental apparatus and the data reduction procedure, and the results and conclusions of this study are presented.

## GOVERNING EQUATIONS

In order to accurately predict the heat loss through the walls and floor of a sub-surface structure, it is necessary to determine the temperature distribution in the surrounding soil. For a one-dimensional (1-D) situation, the unsteady temperature field is governed by

$$\rho_a C_s \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x} \left( k_s \frac{\partial T}{\partial x} \right). \quad (1)$$

Both the apparent thermal conductivity of moist soil,  $k_s$ , and the soil heat capacity,  $C_s$ , are functions of the soil temperature and moisture content. Therefore, in order to determine the temperature distribution, it is necessary to obtain the soil moisture content.

The unsteady 1-D soil moisture distribution is described by

$$\rho_a \frac{\partial W}{\partial \tau} = \frac{\partial}{\partial x} \left( D^* \frac{\partial T}{\partial x} + K \frac{\partial W}{\partial x} \right), \quad (2)$$

where  $D^*$  and  $K$  are the thermal and moisture diffusion coefficients, respectively. If the diffusion coefficients  $D^*$  and  $K$  are known, equations (1) and (2) can be solved simultaneously to obtain the temperature and moisture distributions in the soil.

When unsaturated soil, initially at a uniform moisture content, is subjected to a temperature gradient, moisture transfer occurs due to water flow in both liquid and vapor phases. For a 1-D situation, the mass flux of liquid water,  $J_l$ , and the mass flux of water vapor,  $J_v$ , can be expressed as

$$J_l = -\left(D_l^* \frac{dT}{dx} + K_l \frac{dW}{dx}\right), \quad (3)$$

$$J_v = -\left(D_v^* \frac{dT}{dx} + K_v \frac{dW}{dx}\right). \quad (4)$$

An expression for the net moisture flux can be obtained by combining equations (3) and (4)

$$J_{\text{net}} = J_l + J_v = -\left(D_l^* \frac{dT}{dx} + K_l \frac{dW}{dx}\right) - \left(D_v^* \frac{dT}{dx} + K_v \frac{dW}{dx}\right),$$

or

$$J_{\text{net}} = -(D_l^* + D_v^*) \frac{dT}{dx} - (K_l + K_v) \frac{dW}{dx}. \quad (5)$$

By combining the vapor and liquid diffusion coefficients

$$D^* = D_l^* + D_v^*,$$

$$K = K_l + K_v,$$

equation (5) can be written as

$$J_{\text{net}} = -D^* \frac{dT}{dx} - K \frac{dW}{dx}. \quad (6)$$

Since both  $D^*$  and  $K$  are physical properties, they should be independent of time. Therefore, the diffusion coefficients determined at any specific instant (e.g. quasi steady-state) must be applicable through the process. Under steady-state conditions, the net moisture flux is zero, and equation (6) can be written as

$$\frac{D^*}{K} = -\frac{dW/dx}{dT/dx}. \quad (7)$$

Equation (7) indicates that the ratio of the diffusion coefficients,  $D^*/K$ , can be evaluated from measurements of the moisture and temperature distributions in a soil sample in which 1-D steady-state conditions exist. Although the ratio of  $D^*$  to  $K$  is useful, it is necessary to obtain individual values of  $D^*$  and  $K$  in order to solve equations (1) and (2).

Based on experiments performed on various types of soils, previous researchers have shown that the mass flux of liquid water is predominantly due to a moisture gradient; i.e.  $D_l^*$  is negligible in equation (3). Therefore

$$D^* \simeq D_v^*,$$

and  $D_v^*$  can be estimated by starting with Fick's diffusion equation for mass flux of water vapor into air due to a moisture gradient and then modifying it to account for the presence of soil particles and the possibility that the pores between the particles may be partially filled with liquid water. The resulting expression for the thermal diffusion coefficient of water vapor in moist soil,  $D_v^*$ , is

$$D_v^* = (\rho_a + \rho_v) f \varepsilon D \frac{1}{1-w} \frac{dw}{dT}. \quad (8)$$

The porosity  $f$ , as used in equation (8), is defined as the ratio of the volume of pores not filled with liquid water to the total volume of the moist soil. For non-swelling soils,  $f$  can be expressed in terms of the liquid and soil densities and the saturation and existing moisture contents by

$$f = \frac{\rho_d}{\rho_l} (W_s - W). \quad (9)$$

The tortuosity  $\varepsilon$  accounts for the non-straight line path travelled by the water vapor, and is defined as the ratio of the straight line path to the actual path length travelled by the water vapor. For the experiments of this study,  $\varepsilon$  was assumed to be 0.65. For the condition that some liquid water coexists with water vapor at all locations in the soil, the mass fraction of water vapor,  $w$ , will be equal to the saturation value and will only be a function of temperature and pressure. Experimental data for the diffusion coefficient of water vapor into air,  $D$ , have been reported in ref. [6]. Semi-empirical relationships presented in refs. [6–8] indicate that  $D$  varies linearly with temperature. In the present investigation the following linear curve fit of the experimental data was used to estimate  $D$  for use in equation (8)

$$D (\text{cm}^2 \text{ s}^{-1}) = 1.7255 \times 10^{-3} (T + 273) - 0.2552, \quad (10)$$

where,  $T$  is in  $^{\circ}\text{C}$ .

Equation (8) indicates that the thermal diffusion coefficient of water vapor in moist soil,  $D_v^*$ , is a function of both temperature and soil moisture content. An examination of  $D_v^*$  as a function of temperature with moisture content as a parameter and as a function of moisture content with temperature as a parameter reveals that the thermal diffusion coefficient of water vapor in moist soils,  $D_v^*$ , decreases with both decreasing temperature and increasing moisture content.

The intent of the present investigation was to determine  $D^*$  and  $K$  by designing an apparatus in which 1-D steady-state temperature and moisture fields can be established. Ratios of the diffusion coefficients were determined by first measuring the temperature and moisture distributions in the soil samples and then using these distributions to establish values for  $dT/dx$  and  $dW/dx$  for substitution into equation (7). Next, equation (8) was used to estimate the thermal diffusion coefficient of water vapor in moist

soils. The moisture diffusion coefficient,  $K$ , was then determined by combining the experimentally determined values of  $D^*/K$ , the estimates for  $D^*$ , and the approximation that  $D^* = D_v^*$ .

#### EXPERIMENTAL APPARATUS AND PROCEDURE

An impermeable soil-filled cylinder with perfect insulation around its circumference and fixed end temperatures would provide the ideal conditions for establishing 1-D steady-state heat and moisture transfer. Thus, an apparatus was designed to closely approximate the ideal case. The general design approach was to first determine the temperature profile that would exist in an idealized cylindrical soil sample with constant end temperatures and an assumed axially varying moisture content (thus defining the thermal conductivity distribution [9]). This profile thus became a baseline for comparison. Next, a number of practical apparatus configurations, consisting of soil contained in a CPVC tube surrounded by insulation, were analyzed for the same assumed axial moisture distribution as that used for the idealized case. The resulting temperature distributions were compared to the baseline profile in order to determine the apparatus configuration which closely approximated 1-D heat transfer in the soil. The computed temperature distributions, as a function of insulation thickness and thermal conductivity, were found to be substantially different from the baseline profile, thus indicating the existence of two-dimensional (2-D) heat transfer. The analysis was then performed for a design that would use a thin metal sheet around the outside of the insulation. This sheet of metal served as a guard heater in that the combination of conduction along the metal and convection to the room air provided an axially varying temperature boundary on the outer surface of the insulation, thus aiding in the establishment of 1-D heat transfer within the soil. Details of the apparatus configuration and the design process are presented in ref. [10].

A schematic of the apparatus identifying the main components, is presented in Fig. 1. It consists of a soil sample contained in a CPVC tube with one end (the warm end) attached to an insulated copper plate equipped with a silicone rubber stock heater. The other end of the cylinder (the cold end) was attached to an uninsulated copper plate, thus providing an impermeable boundary at room temperature of approximately 20°C. Thermocouples were located approximately along the axis of symmetry within the soil and in the warm and cold end plates. No attempt was made to measure the radial temperature gradients within the soil or the surrounding insulation.

In order to make an experimental run, one end of the CPVC tube was first attached to the warm end plate. Soil, with a predetermined moisture content, was then packed into the tube at uniform density. The cold end plate was then attached to prevent moisture loss. Next, the thermocouples, the insulation, and the thin metal

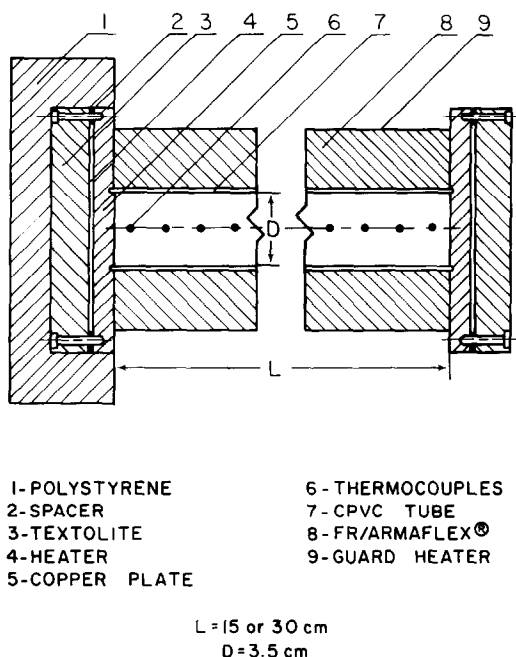


FIG. 1. Schematic of the experimental apparatus.

sheet were successively installed. The heater was then connected to a temperature controller and the entire warm end was maintained at approximately 40°C.

Temperature profiles were recorded during all the experiments. At the end of a run, the apparatus was disassembled and approximately 1 cm long soil samples were removed from the CPVC tube and placed in aluminum containers with tight fitting lids. The samples were weighed, dried in an oven at  $110 \pm 5^\circ\text{C}$  for 24 h, and then reweighed. The difference between the mass of the moist and the dry soil samples provided a profile of the mass of liquid water for the experiment. The moisture content distribution was then calculated as the ratio of the mass of liquid water to the mass of dry soil. (The procedure for determining the soil moisture content was conducted in accordance with ASTM Standard D-2216.)

#### RESULTS AND DISCUSSION

Two types of soil, which have been designated as 'In-situ' and 'Waukegan', were studied. Their texture and particle size distribution are listed in Table 1.

Table 1. Particle size distribution of the soils

|                                           | In-situ soil | Waukegan soil |
|-------------------------------------------|--------------|---------------|
| Gravel and sand ( $> 0.074 \text{ mm}$ )  | 88.7%        | 28.0%         |
| Silt ( $0.005\text{--}0.074 \text{ mm}$ ) | 8.2%         | 61.0%         |
| Clay ( $< 0.005 \text{ mm}$ )             | 3.1%         | 11.0%         |
| Soil classification                       | Sand         | Silt loam     |

Table 2. List of experiments

| Soil type<br>$W_s$<br>Group | I    |      |      | II   |      |      | III  |      |      |      | Waukegan<br>0.370<br>IV |      |      |
|-----------------------------|------|------|------|------|------|------|------|------|------|------|-------------------------|------|------|
| Experiment No.              | 1    | 2    | 3    | 10   | 12   | 13   | A    | B    | C    | D    | 4                       | 5    | 11   |
| Length, cm                  | 30   | 30   | 30   | 15   | 15   | 15   | 30   | 30   | 30   | 30   | 30                      | 30   | 15   |
| $W_i/W_s$ , %               | 21   | 21   | 21   | 21   | 20   | 20   | 11   | 17   | 33   | 42   | 48                      | 25   | 22   |
| Duration, days              | 24   | 58   | 80   | 20   | 6    | 15   | 29   | 24   | 20   | 20   | 18                      | 25   | 20   |
| $T_H$ , °C                  | 39.9 | 40.5 | 40.1 | 39.8 | 40.0 | 40.0 | 40.1 | 40.1 | 39.9 | 41.0 | 40.4                    | 40.0 | 40.0 |
| $T_C$ , °C                  | 20.6 | 20.5 | 20.3 | 20.5 | 21.4 | 20.9 | 20.6 | 22.9 | 21.2 | 20.7 | 19.6                    | 20.1 | 21.0 |
| Water loss, † %             | 5    | 10   | 10   | 16   | 10   | 13   | 1    | 1    | 4    | 2    | 3                       | 6    | 10   |

† This is the relative difference between the average moisture content between the beginning and the end of the experiment.

Experiments were performed on 15 and 30 cm long soil samples with end temperatures of approximately 40 and 20°C. In order to conveniently present and compare the results, the experiments are divided into four groups as listed in Table 2. Group I experiments, which required approximately 24 days to achieve thermal steady-state, were performed on 30 cm long samples of In-situ soil with similar initial moisture contents for the indicated durations. Group II experiments were conducted on 15 cm long samples of In-situ soil with initial and boundary conditions similar to those for Group I but with run periods approximately one-quarter those of Group I in order to determine the effect of sample length on the time required to achieve steady-state moisture distribution. Group III consists of experiments on 30 cm long samples of In-situ soil with varying initial moisture contents. The runs in this group were terminated after thermal steady-state had been achieved. Experiments on 15 and 30 cm long samples of Waukegan soil are presented as Group IV.

#### Temperature distributions

Experimentally it has been shown that the thermal conductivity of moist soil is primarily a function of the soil moisture content and composition [9]. Hence, if the thermal conductivity of a moist soil sample were

to remain uniform when subjected to a temperature difference, then a linear temperature distribution would be expected under steady-state conditions. Since the heat and mass transfer processes are coupled, the steady-state temperature profile will be non-linear because of the moisture redistribution that results from the applied temperature difference. Transient temperature measurements, presented in Fig. 2 for one of the experiments, confirms this process. The figure presents the temperature profiles that exist for various times after the temperature at one end of the sample was suddenly increased. (Prior to that time the entire sample was at the cold end temperature.) It is seen that the soil temperature increased, approached a linear distribution (because of the initially uniform moisture content), and thereafter decreased to a steady-state non-linear profile.

The steady-state temperature distributions for experiments on the 30 cm long samples of In-situ soil (Groups I and III) and on the Waukegan soil (Group IV) are shown in Figs. 3 and 4, respectively. Since the slopes of these temperature distributions decrease with axial distance from the warm end ( $x/L$ ), the profiles display the characteristics of a material whose thermal conductivity increases with  $x/L$ . The figures also indicate that the shapes of the temperature profiles are a function of the initial moisture content. An exami-

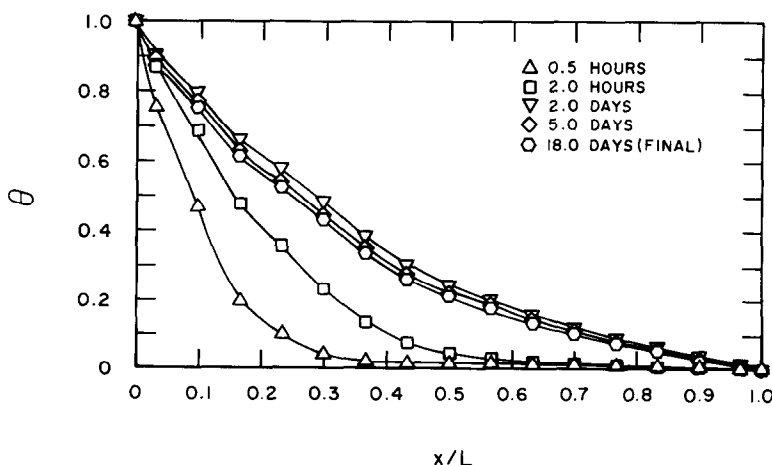


FIG. 2. Transient and steady-state temperature profiles (Waukegan soil,  $L = 30$  cm,  $W_i/W_s = 0.48$ ).

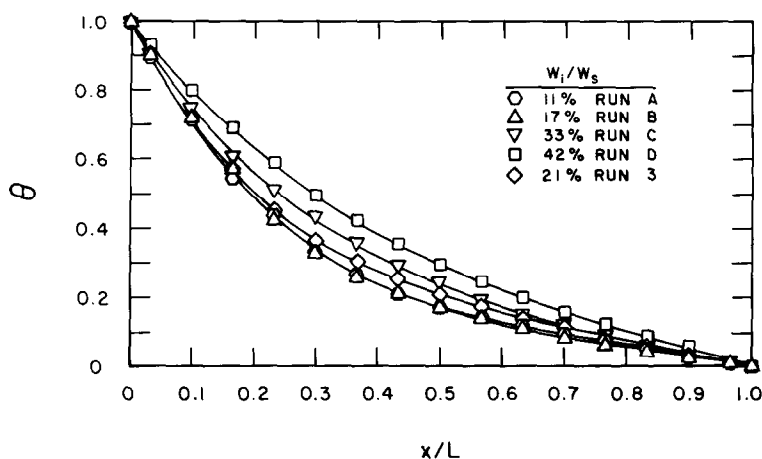


FIG. 3. Steady-state temperature distributions in 30 cm long samples of In-situ soil—Groups I and III.

nation of Fig. 3 reveals that the deviation from a linear temperature profile decreases as the initial moisture content increases. The same trend exists for the two tests conducted on similar lengths of Waukegan soil (Fig. 4).

#### Moisture content profiles

Figures 5–7 present the moisture content,  $W/W_s$ , as a function of the axial distance from the warm end,  $x/L$ . Figure 5 shows the transient nature of moisture transfer for both the 30 and 15 cm long samples of In-situ soil (Groups I and II). In all of these experiments the initial moisture content was approximately the same ( $\approx 21\%$  of saturation). Attention is first directed to the curves for experiments using the 30 cm long samples. Changes in the moisture distribution are seen to occur between the 24 and 58 day runs, but no significant changes are observed between the 58 and 80 day runs. Next, attention is drawn to the experiments performed on the 15 cm long samples. Dimensional analysis of the heat and mass transfer process indicates that the time required to reach steady-state moisture distribution is proportional to the square of the sample length [11].

Therefore, the experiments using a 15 cm long sample are expected to reach steady-state moisture distribution in one-quarter of the time as that for the 30 cm long samples. For the 15 cm long samples, the shape of the profile changes between 6 and 15 days, but no significant change is observed between 15 and 20 days. This result is consistent with that of the 30 cm tests where changes continued after 24 days but not after 58 days. However, a comparison of the profiles for the long and short test sections reveals substantial differences.

Moisture distributions for the 30 cm long samples of In-situ soil with initial moisture contents in the range 11–42% of saturation (Groups I and III) are shown in Fig. 6. (In the case of the 21% moisture content, the 80 day run is presented.) The difference in moisture content between the warm and cold ends is very slight for the case of  $W_i/W_s = 0.42$ . The axial variation becomes more pronounced as the initial moisture content is reduced to 33 and 21% of saturation. For the case of  $W_i/W_s = 0.21$ , a very dry region occurs near the warm end followed by a sudden rise in moisture content and then the curve becomes relatively flat. No significant changes in moisture content occur at the

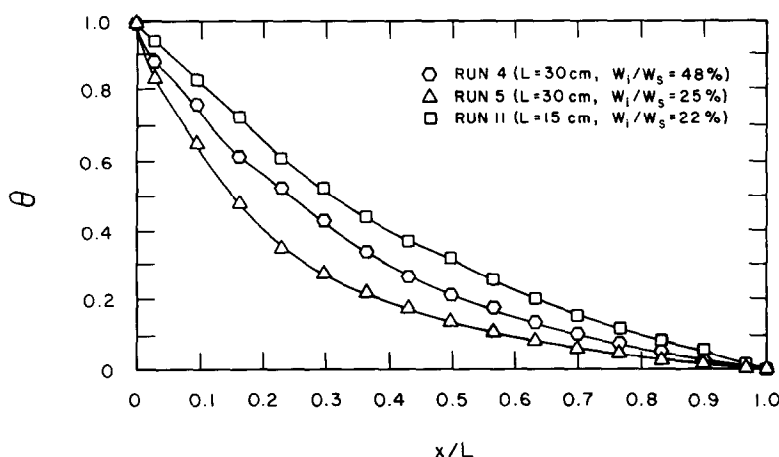


FIG. 4. Steady-state temperature profiles in the Waukegan soil—Group IV.

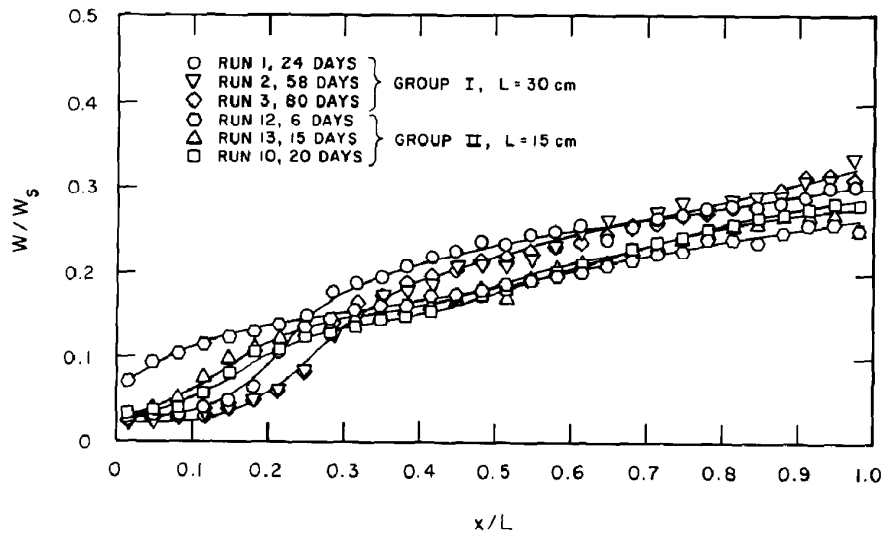


FIG. 5. Distribution of moisture in 30 and 15 cm long samples of In-situ soil—Groups I and II,  $W_i/W_s = 0.21$ .

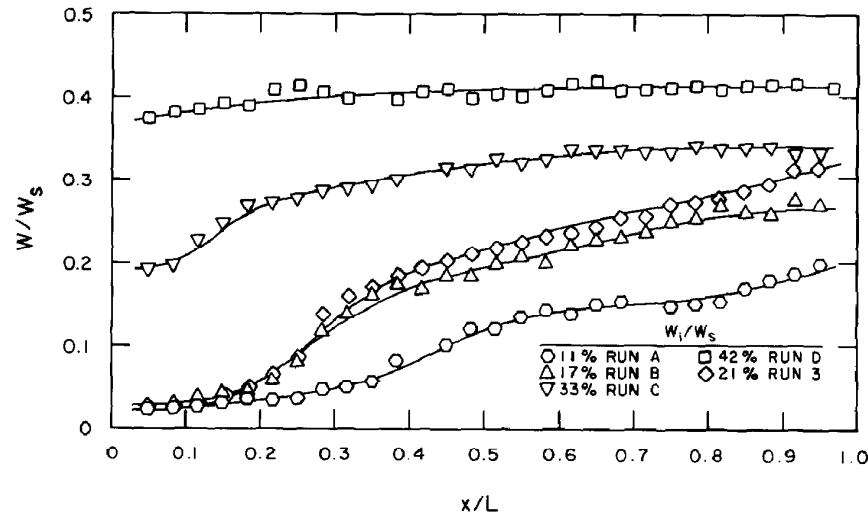


FIG. 6. Moisture content profiles in 30 cm long samples of In-situ soil—Groups I and III.

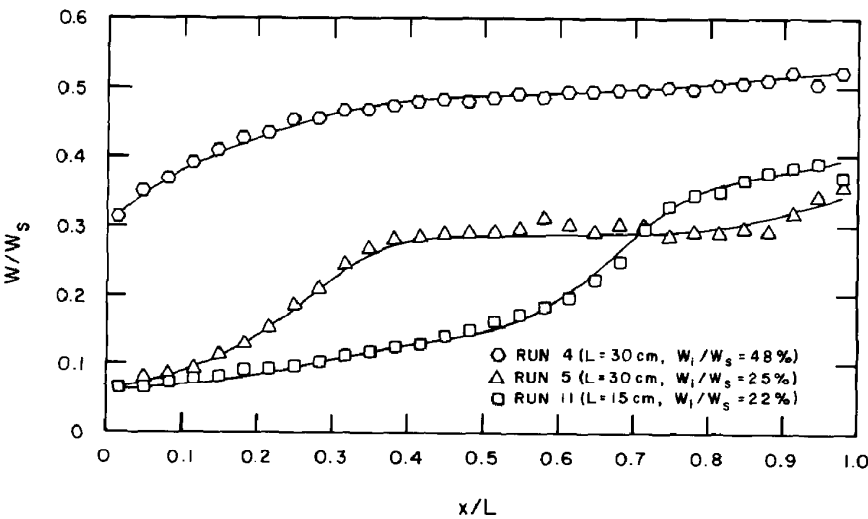


FIG. 7. Moisture content distributions in the Waukegan soil—Group IV.

warm end when  $W_i/W_s$  is reduced below 21% of saturation. As a result, the axial variation in moisture content decreases with further reductions in the initial moisture content. Similar trends have been observed by other investigators [12, 13]. The initial moisture content at which the maximum moisture transfer due to a temperature gradient occurs was designated by Bouyoucos [12] as the 'thermal critical moisture content'. Based on the experimental data presented in Fig. 6, the 'thermal critical moisture content' for the In-situ soil appears to be in the neighbourhood of 21% of saturation.

The steady-state moisture distributions for experiments on the 15 and 30 cm long samples of Waukegan soil (Group IV) are presented in Fig. 7. Although the general trends are similar to those for the In-situ soil, a comparison of Figs. 5 and 7 reveals the following interesting, and as yet unexplained, differences in the profiles for the short and long test sections of the two soils with nearly similar initial moisture content: (1) for the shorter sample of the Waukegan soil, the sharp rise in moisture content occurs at a substantially greater value of  $x/L$ ; and (2) at the cold end ( $x/L = 1$ ), the In-situ soil's moisture content ( $W/W_s$ ) for the 15 cm long sample is less than that for the 30 cm long sample (Fig. 5), while just the opposite is true for the Waukegan soil (Fig. 7).

The average moisture content within the test cell at the end of an experiment was evaluated by integrating the moisture profile. A comparison of this value with the initial moisture content provided a measure of how much moisture was lost during the experiment. This relative water loss is included in Table 2. In all cases, this loss was less than 16%.

#### Diffusion coefficients

The results of the temperature and moisture content profiles were used to obtain the necessary inputs for equation (7) to calculate  $D^*/K$  as a function of axial

distance from the warm end (in increments of 0.02). An example of these results is shown in Fig. 8. A more useful form of the results is to obtain  $D^*/K$  as a function of the soil moisture content. These can be obtained by combining the  $D^*/K$  vs  $x/L$  and the  $W/W_s$  vs  $x/L$  information. Figures 9 and 10 present  $D^*/K$  vs  $W/W_s$  for the In-situ and the Waukegan soils, respectively. The basic trends are similar for both soils. The ratio of diffusion coefficients,  $D^*/K$ , is small for low moisture content. As the moisture content increases, the ratio also increases and reaches a broad maximum at a certain value of  $W/W_s$  which depends on the soil type. Further increases in moisture content results in a decrease in the ratio of the diffusion coefficients. The decrease in  $D^*/K$  at high moisture contents is expected since the soil porosity decreases causing a reduction in the amount of vapor transfer. Based on his pioneering work in the field of combined heat and mass transfer in porous material, Luikov [14] has shown that  $D^*/K$  approaches zero as the soil becomes saturated ( $W/W_s \rightarrow 1.0$ ). In the dry region of the soil,  $D^*/K$  should also decrease to a small value, since mass transfer of water vapor is not possible due to the absence of liquid water. The experimental data presented by Luikov [14] indicates that  $D^*/K$  is approximately zero for  $W/W_s = 0$ .

In Figs. 9 and 10, no attempt has been made to display the effect of temperature on  $D^*/K$ . An analysis of the data with regard to temperature revealed that for the investigated range of temperature and moisture content, the  $D^*/K$  vs  $W/W_s$  results were insensitive to temperature.

In Fig. 9, it is interesting to note that all the Group I and Group II data are included. In spite of the fact that there was a significant difference in the moisture distribution between the short and long duration runs (24 days vs 58 days for the 30 cm case and 6 days vs 15 days for the 15 cm case), no noticeable differences appear in the  $D^*/K$  values. It appears that after the initial period, during which temperature changes are

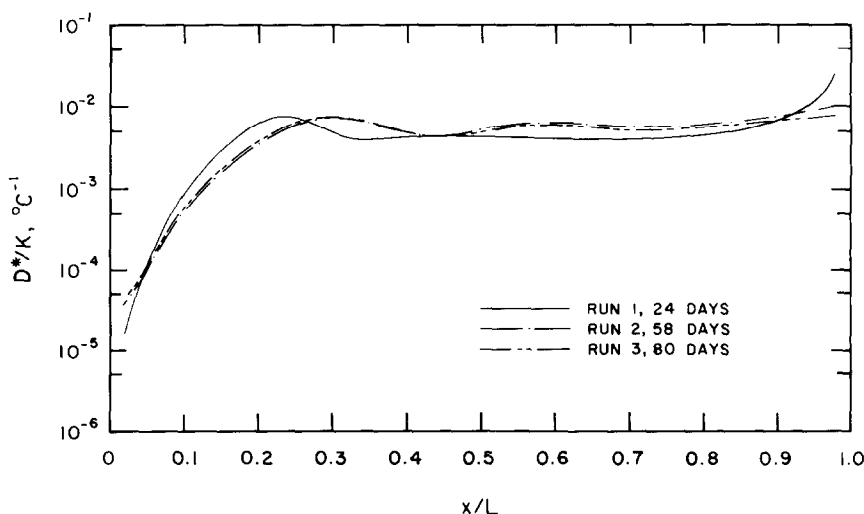


Fig. 8. Distributions of the ratio of diffusion coefficients for Group I (In-situ soil,  $L = 30$  cm,  $W_i/W_s = 0.21$ ) experiments.



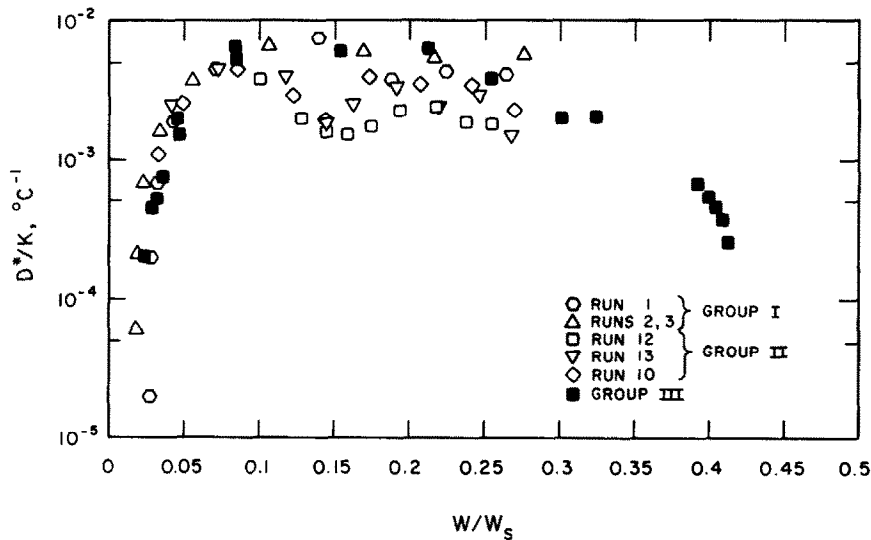


FIG. 9. Ratio of the diffusion coefficients as a function of moisture content for the In-situ soil—Groups I–III.

easily detected, the changes in both temperature and moisture content occur slowly enough that it may be possible to treat the problem as quasi steady-state.

Equation (7) indicates that the ratio of the diffusion coefficients,  $D^*/K$ , is a function of the gradients of the temperature distributions and the moisture content profiles. Figures 3–7 show that curve-fits and slopes for the temperature distributions are more easily obtainable than for the soil moisture content profiles. In the present investigation, the cubic spline technique was used to obtain the curve-fits shown in Figs. 5–7 and to obtain  $dW/dx$  values for use in equation (7). The sensitivity of  $D^*/K$  to the slopes of the moisture content profiles is revealed in Fig. 9 by a dip or valley in  $D^*/K$  in the neighborhood of  $W/W_s = 0.15$ . It is interesting to note that all of the data points in the  $D^*/K$  valley belong to Group II. For these experiments, on the 15 cm long samples, Fig. 5 shows a small dip in the moisture content distribution for  $x/L$  between 0.3 and 0.6. This

dip affects the  $dW/dx$  values in this region and hence causes the aforementioned  $D^*/K$  valley.

Based on theoretical analysis and experimental data, Luikov [15] has proposed that for the coupled heat and mass transfer process in a porous material, the ratio of the diffusion coefficients should be less than  $1.0 \times 10^{-2} \text{ } ^\circ\text{C}^{-1}$ . The data presented in Fig. 9 indicates that for the In-situ soil,  $D^*/K$  has a maximum value of approximately  $0.75 \times 10^{-2} \text{ } ^\circ\text{C}^{-1}$  in the region of  $W/W_s = 0.15$ . For the Waukegan soil, the maximum  $D^*/K$  value of approximately  $2.0 \times 10^{-2} \text{ } ^\circ\text{C}^{-1}$  occurs in the region of  $W/W_s = 0.23$  (Fig. 10). The differences in the maximum values of  $D^*/K$  and in the  $W/W_s$  regions where this occurs may be a result of the soil's clay content (3.1% in the In-situ soil compared to 11% in the Waukegan soil).

Predicted values of  $D^*$  [equation (8)], and the  $D^*/K$  data were used to estimate the moisture diffusion coefficient,  $K$ , as a function of moisture content. Figures

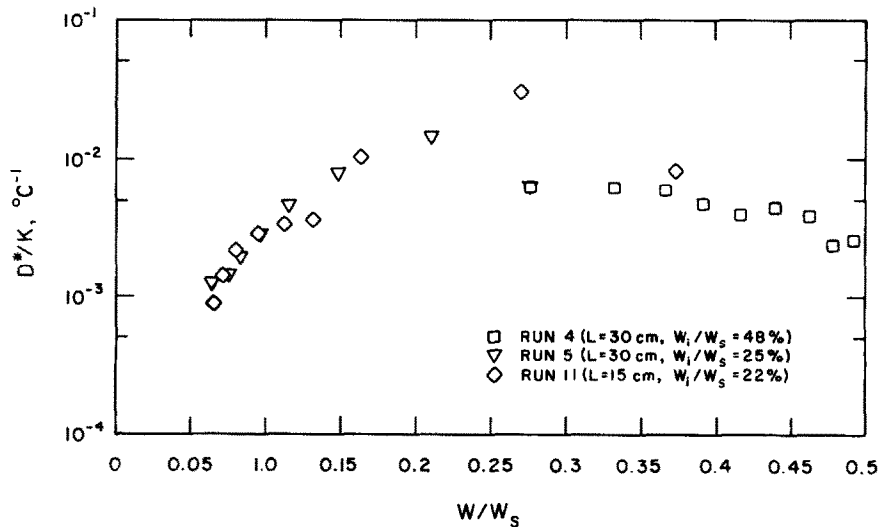


FIG. 10. Ratio of the diffusion coefficients as a function of moisture content for the Waukegan soil—Group IV.

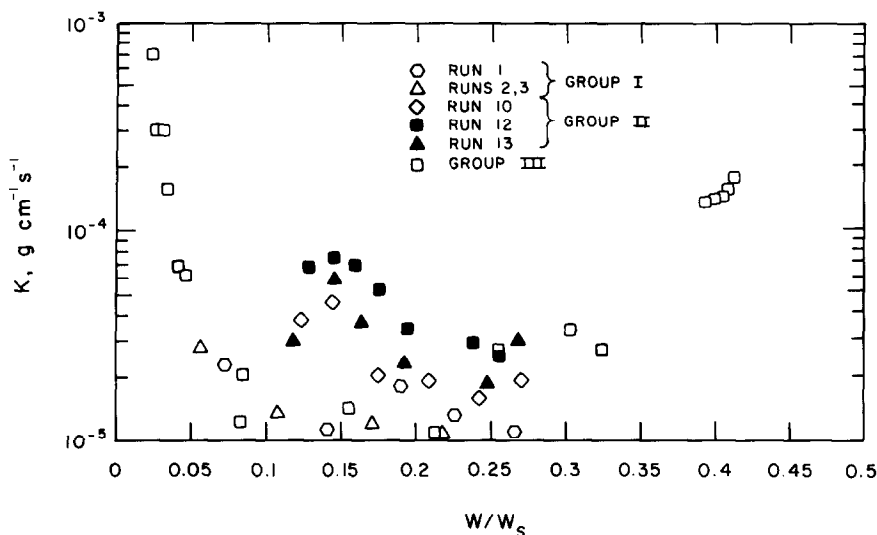


FIG. 11. Moisture diffusion coefficient as a function of moisture content for the In-situ soil—Groups I–III.

11 and 12 show the variation of  $K$  with  $W/W_s$  for the In-situ and Waukegan soils, respectively. The moisture diffusion coefficient exhibits a broad minimum since it is inversely proportional to  $D^*/K$  and since  $D^*$  varies monotonically with  $W/W_s$ . For the soils tested, the value of  $K$  is around  $10^{-5}$ – $10^{-4}$   $\text{g s}^{-1} \text{cm}^{-1}$ .

#### CONCLUSIONS

Ratios of the diffusion coefficients,  $D^*/K$ , were experimentally determined for two types of soils by measuring 1-D temperature and moisture distributions in 15 and 30 cm long cylindrical soil samples for temperatures in the range of 20–40°C. Transient measurements revealed that temperature adjusts rapidly compared to moisture, and that quasi steady-state exists during the time of moisture redistribution. The time required to achieve steady-state moisture distribution is relatively long; however, reliable

estimates of  $D^*/K$  can be obtained by concluding the experiments after the quasi steady-state condition has been achieved.

The steady-state moisture and temperature profiles were found to be functions of both the initial moisture content and the soil type. In general, an increase in the initial moisture content leads to a uniform moisture distribution, thus a uniform thermal conductivity and a nearly linear temperature profile.

The ratio of the diffusion coefficients,  $D^*/K$ , was found to be a function of both the soil composition and the soil moisture content. The ratio was found to be small for low moisture content values, increase with moisture content and attain a broad maximum in certain regions of  $W/W_s$  (determined by the soil composition), and thereafter decrease with further increases in  $W/W_s$ . Except for the drier regions near the warm end,  $D^*/K$  was found to be in the range of  $10^{-3}$ – $10^{-2} \text{C}^{-1}$ . For the In-situ soil,  $D^*/K$  was below this

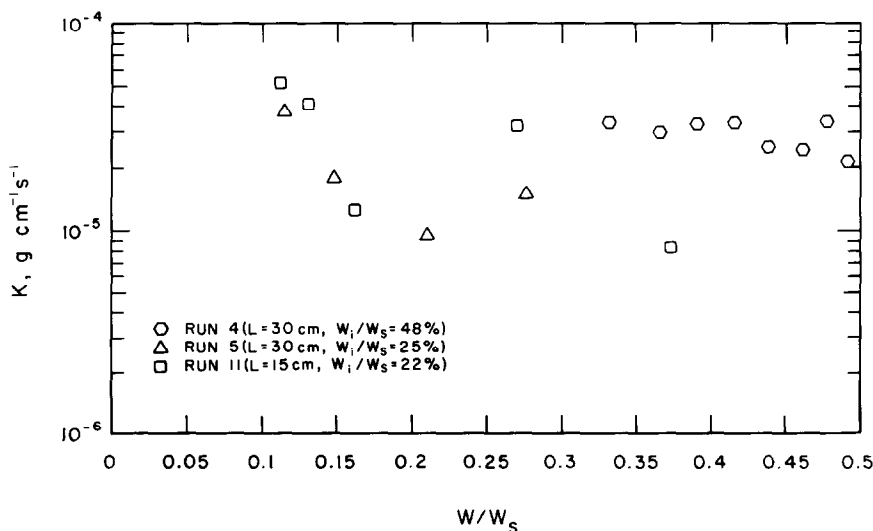


FIG. 12. Moisture diffusion coefficient as a function of moisture content for the Waukegan soil—Group IV.

range for  $W/W_s$  less than approximately 0.04 (near the warm end) and  $W/W_s$  higher than 0.35 (near the cold end). The thermal diffusion coefficient of water vapor in moist soil,  $D_v^*$ , was predicted to decrease with both decreasing temperature and increasing moisture content. For the soils used in this study, the moisture diffusion coefficient,  $K$ , was estimated to be in the range  $10^{-5}$ – $10^{-4}$  g s<sup>-1</sup> cm<sup>-1</sup>.

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#### DETERMINATION EXPERIMENTALE DES COEFFICIENTS DE TRANSFERT DE CHALEUR ET DE MASSE DANS LES SOLS HUMIDES NON SATURES

**Résumé**—Les rapports du coefficient de diffusion thermique au coefficient de diffusion d'humidité sont déterminés expérimentalement pour deux types de sol. Le rapport des coefficients de diffusion croît avec le contenu d'humidité, atteint un maximum et décroît ensuite. Excepté pour les régions les plus sèches proches de l'extrémité chaude, le rapport est dans le domaine  $10^{-3}$ – $10^{-2}$  °C<sup>-1</sup>. Des précisions analytiques pour les coefficients de diffusion thermique sont combinés avec les résultats expérimentaux, pour déterminer les coefficients de diffusion de l'humidité. Le coefficient de diffusion thermique est calculé en décroissance avec à la fois la température décroissante et le contenu en humidité croissant. Pour les sols considérés dans cette étude, le coefficient de diffusion d'humidité est estimé dans le domaine  $10^{-5}$ – $10^{-4}$  g s<sup>-1</sup> cm<sup>-1</sup>.

#### EXPERIMENTELLE BESTIMMUNG VON WÄRME- UND STOFFTRANSPORTKOEFFIZIENTEN IN FEUCHTEN, UNGESÄTTIGTEN BÖDEN

**Zusammenfassung**—Für zwei Bodensorten wurde experimentell das Verhältnis eines Transportkoeffizienten für thermisch bedingte Diffusion und eines Feuchteleitkoeffizienten bestimmt. Es zeigte sich, dass das Verhältnis der Transportkoeffizienten mit zunehmendem Feuchtigkeitsgehalt zunächst zunimmt, ein breites Maximum erreicht und dann abnimmt. Mit Ausnahme der trockenen Gebiete am warmen Ende lag das Verhältnis im Bereich  $10^{-3}$  bis  $10^{-2}$  °C<sup>-1</sup>. Zur Bestimmung des Feuchteleitkoeffizienten wurden die experimentellen Ergebnisse mit Rechenwerten für den 'thermischen Diffusionskoeffizienten' kombiniert. Die Berechnung des 'thermischen Diffusionskoeffizienten' ergab eine Abnahme mit abnehmender Temperatur und mit zunehmendem Feuchtigkeitsgehalt. Für die hier untersuchten Böden wurden die Feuchteleitkoeffizienten zu  $10^{-5}$  bis  $10^{-4}$  g s<sup>-1</sup> cm<sup>-1</sup> abgeschätzt.

#### ЭКСПЕРИМЕНТАЛЬНОЕ ОПРЕДЕЛЕНИЕ КОЭФФИЦИЕНТОВ ТЕПЛО-И МАССОПЕРЕНОСА ВЛАЖНЫХ НАСЫЩЕННЫХ ПОЧВ

**Аннотация**—Измерены отношения коэффициента диффузии тепла и влаги для двух видов почв. Это отношение увеличивается с ростом влагосодержания, достигает устойчивого максимума, а затем снижается. За исключением более сухих мест теплоподвода величина отношения лежит в диапазоне  $10^{-3}$ – $10^{-2}$  °C<sup>-1</sup>. Расчетные величины коэффициентов термической диффузии согласуются с измерениями коэффициентов диффузии влаги. Показано, что коэффициент термической диффузии снижается при понижении температуры и уменьшении влагосодержания. Для исследованных почв величина коэффициента диффузии влаги находится в диапазоне  $10^{-5}$ – $10^{-4}$  сек<sup>-1</sup> см<sup>-1</sup>.